

Soft-Gluon Resummation and PDF Theory Uncertainties[†]

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Abstract

Parton distribution functions are determined by the comparison of finite-order calculations with data. We briefly discuss the interplay of higher order corrections and PDF determinations, and the use of soft-gluon resummation in global fits.

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Soft-Gluon Resummation and PDF Theory Uncertainties

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Parton distribution functions are determined by the comparison of finite-order calculations with data. We briefly discuss the interplay of higher order corrections and PDF determinations, and the use of soft-gluon resummation in global fits.

1. FACTORIZATION & THE NLO MODEL

A generic inclusive cross section for the process $A + B \rightarrow F + X$ with observed final-state system F , of total mass Q , can be expressed as

$$Q^4 \frac{d\sigma_{AB \rightarrow FX}}{dQ^2} = \phi_{a/A}(x_a, \mu^2) \otimes \phi_{b/B}(x_b, \mu^2) \otimes \hat{\sigma}_{ab \rightarrow FX}(z, Q, \mu), \quad (1)$$

with $z = Q^2/x_a x_b S$. The $\hat{\sigma}_{ab}$ are partonic hard-scattering functions, $\hat{\sigma} = \sigma_{\text{Born}} + (\alpha_s(\mu^2)/\pi)\hat{\sigma}^{(1)} + \dots$. They are known to NLO for most processes in the standard model and its popular extensions. Corrections begin with higher, uncalculated orders in the hard scattering, which respect the form of Eq. (1). The discussion is simplified in terms of moments with respect to $\tau = Q^2/S$,

$$\begin{aligned} \tilde{\sigma}_{AB \rightarrow FX} &= \int_0^1 d\tau \tau^{N-1} Q^4 d\sigma_{AB \rightarrow FX}/dQ^2 \\ &= \sum_{a,b} \tilde{\phi}_{a/A}(N, \mu^2) \tilde{\sigma}_{ab \rightarrow FX}(N, Q, \mu) \tilde{\phi}_{b/B}(N, \mu^2), \quad (2) \end{aligned}$$

where the moments of the ϕ 's and $\hat{\sigma}_{ab \rightarrow FX}$ are defined similarly.

Eqs. (1) and (2) are starting-points for both the determination and the application of parton distribution functions (PDFs), $\phi_{i/H}$, using 1-loop $\hat{\sigma}$'s [1–3]. We may think of this collective enterprise as an “NLO model” for the PDFs, and for hadronic hard scattering in general. For precision applications we ask how well we really know the PDFs [4–6]. Partly this is a question of how well data constrain them, and partly it is a question of how well we *could* know them, given finite-order calculations in Eqs. (1) and (2). We will not attempt here to assign error estimates to theory. We hope, however, to give a sense of how to distinguish ambiguity from uncertainty, and how our partial knowledge of higher orders can reduce the latter.

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2. UNCERTAINTIES, SCHEMES & SCALES

It is not obvious how to quantify a “theoretical uncertainty”, since the idea seems to require us to estimate corrections that we haven’t yet calculated. We do not think an unequivocal definition is possible, but we can try at least to clarify the concept, by considering a hypothetical set of nucleon PDFs determined from DIS data alone [4]. To make such a determination, we would invoke isospin symmetry to reduce the set of PDF’s to those of the proton, $\phi_{a/P}$, and then measure a set of singlet and nonsinglet structure functions, which we denote $F^{(i)}$. Each factorized structure function may be written in moment space as

$$\tilde{F}^{(i)}(N, Q) = \sum_a \tilde{C}_a^{(i)}(N, Q, \mu) \tilde{\phi}_{a/P}(N, \mu^2), \quad (3)$$

in terms of which we may solve for the parton distributions by inverting the matrix \tilde{C} ,

$$\tilde{\phi}_{a/P}(N, \mu^2) = \sum_i \tilde{C}^{-1(i)}_a(N, Q, \mu) \tilde{F}^{(i)}(N, Q). \quad (4)$$

With “perfect” \tilde{F} ’s at fixed Q , and with a specific approximation for the coefficient functions, we could solve for the moment-space distributions numerically, without the need of a parameterization. In a world of perfect data, but of incompletely known coefficient functions, uncertainties in the parton distributions would be entirely due to the “theoretical” uncertainties of the C ’s:

$$\delta \tilde{\phi}_{a/P}(N, \mu) = \sum_i \delta \tilde{C}^{-1(i)}_a(N, Q, \mu) \tilde{F}^{(i)}(N, Q). \quad (5)$$

Our question now becomes, how well do we know the C ’s? In fact this is a subtle question, because the coefficient functions depend on choices of scheme and scale.

Factorization schemes are procedures for defining coefficient functions perturbatively. For example, choosing for F_2 the LO (quark) coefficient function in Eq. (4) defines a DIS scheme (with \tilde{C} independent of μ , which is then to be taken as Q in $\tilde{\phi}$). Computing the C ’s from partonic cross sections by minimal subtraction to NLO defines an NLO $\overline{\text{MS}}$ scheme, and so on.

Once the choices of C 's and μ are made, the PDF's are defined uniquely.

Evolution in an $\overline{\text{MS}}$ or related scheme, enters through

$$\begin{aligned} \mu \frac{d}{d\mu} \tilde{\phi}_{a/H}(N, \mu^2) &= -\Gamma_{ab}(N, \alpha_s(\mu^2)) \tilde{\phi}_{b/H}(N, \mu^2) \\ \mu \frac{d}{d\mu} \tilde{C}_c^{(i)}(N, Q, \mu) &= \tilde{C}_d^{(i)}(N, Q, \mu) \Gamma_{dc}(N, \alpha_s(\mu^2)). \end{aligned} \quad (6)$$

In principle, by Eq. (6), the scale-dependence of the $C_a^{(i)}$ exactly cancels that of the PDFs in Eq. (3) and, by extension, in Eq. (1). This cancelation, however, requires that each C and the anomalous dimensions Γ be known to all orders in perturbation theory.

To eliminate μ -dependence up to order α_s^{n+1} , we need $\hat{\sigma}$ to order α_s^n and the Γ_{ab} to α_s^{n+1} . One-loop (NLO) QCD corrections to hard scattering require two-loop splitting functions, which are known. The complete form of the NNLO splitting functions, is still somewhere over the horizon [7]. Even when these are known, it will take some time before more than a few hadronic hard scattering functions are known at NNLO.

We can clarify the role of higher orders by relating structure functions at two scales, Q_0 and Q . Once we have measured $F(N, Q_0)$, we may predict $F(N, Q)$ in terms of the relevant anomalous dimensions and coefficient functions by

$$\begin{aligned} F(N, Q) &= F(N, Q_0) e^{\int_{Q_0}^Q \frac{d\mu'}{\mu'} \Gamma(N, \alpha_s(\mu'^2))} \\ &\times \left[\frac{\tilde{C}(N, Q, Q)}{\tilde{C}(N, Q_0, Q_0)} \right]. \end{aligned} \quad (7)$$

This prediction, formally independent of PDFs and independent of the factorization scale, has corrections from the next, still uncalculated order in the anomalous dimension and in the ratio of coefficient functions. The asymptotic freedom of QCD gives a special role to LO: only the one-loop contribution to Γ diverges with Q in the exponent, and contributes to the leading, logarithmic scale breaking. NLO corrections already decrease as the inverse of the logarithm of Q , NNLO as two powers of the log. Thus, the theory is self-regulating towards high energy, where dependence on uncalculated pieces in the coefficients and anomalous dimensions becomes less and less important.

The general successes of the NLO model strongly suggest that relations like (7) are well-satisfied for a wide range of observables and values of N (or x) in DIS and other processes. This does not mean, however, that we have no knowledge of, or use for, information from higher orders. In particular, near $x = 1$ PDFs are rather poorly known [8]. At the same time, the ratio of

C 's depends on N , and if $\alpha_s \ln N$ is large, it becomes important to control higher-order dependence on $\ln N$. This is a task usually referred to as resummation, to which we now turn.

3. RESUMMATION

Let us continue our discussion of DIS, describing what is known about the N -dependence of the coefficient functions C , as a step toward understanding the role of higher orders. Specializing again for simplicity to nonsinglet or valence, the resummed coefficient function may be written as [9,10]

$$\tilde{C}^{\text{res}}(N, Q, \mu) = \tilde{C}_{\text{sub}}^{\text{NLO}}(N, Q, \mu) + C_{\delta}^{\text{DIS}} e^{E_{\text{DIS}}(N, Q, \mu)}, \quad (8)$$

where “sub” implies a subtraction on \tilde{C}^{NLO} to keep \tilde{C}^{res} exact at order α_s , and where C_{δ}^{DIS} corresponds to the NLO N -independent (“hard virtual”) terms. The exponent resums logarithms of N :

$$E_{\text{DIS}}(N, Q, \mu) = \quad (9)$$

$$\int_{Q^2/\bar{N}}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[A(\alpha_s(\mu'^2)) \ln(\bar{N}\mu'^2/Q^2) + B(\alpha_s(\mu'^2)) \right],$$

with $\bar{N} \equiv N e^{\gamma_E}$, and with

$$\begin{aligned} A(\alpha_s) &= \\ &\frac{\alpha_s}{\pi} C_F \left[1 + \frac{\alpha_s}{2\pi} \left(C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_F \right) \right] \\ B(\alpha_s) &= \frac{3}{2} C_F \frac{\alpha_s}{2\pi}. \end{aligned} \quad (10)$$

Eq. (10) is accurate to leading (LL) and next-to-leading logarithms (NLL) in N in the exponent: $\alpha_s^m \ln^{m+1} N$ and $\alpha_s^m \ln^m N$, respectively. The N dependence of the ratio $\tilde{C}_2^{\text{res}}(N, Q, Q)/\tilde{C}_2^{\text{NLO}}(N, Q, Q)$ is shown in Fig. 1, with $Q^2 = 1, 5, 10, 100 \text{ GeV}^2$. At $N = 1$ the ratio is unity. It is less than unity for moderate N , but then begins to rise, with a slope that increases strongly for small Q . At low Q^2 and large N , higher orders can be quite important. What does this mean for PDFs? We can certainly refit PDFs with resummed coefficient functions, and we see that the high moments of such PDFs are likely to be quite different from those from NLO fits.

To get a sense of how such an NLL/NLO- $\overline{\text{MS}}$ scheme might differ from a classic NLO- $\overline{\text{MS}}$ scheme, we resort to a model set of resummed distributions, determined as follows. We define valence PDFs in the resummed scheme by demanding that their contributions to F_2 match those of the corresponding NLO valence PDFs at a fixed $Q = Q_0$, which is ensured by

$$\tilde{\phi}^{\text{res}}(N, Q_0^2) = \tilde{\phi}^{\text{NLO}}(N, Q_0^2) \frac{\tilde{C}_2^{\text{NLO}}(N, Q_0, Q_0)}{\tilde{C}_2^{\text{res}}(N, Q_0, Q_0)}. \quad (11)$$

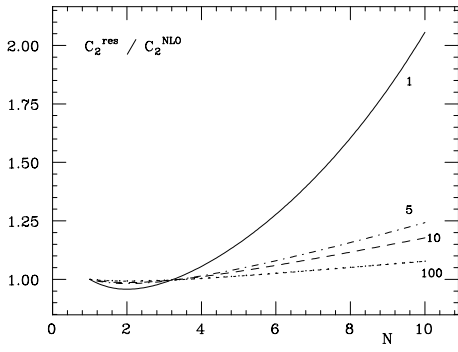


Figure 1. Ratio of Mellin- N moments of resummed and NLO $\overline{\text{MS}}$ -scheme quark coefficient functions for F_2 . The numbers denote the value of Q^2 in GeV^2 . We have chosen $\mu = Q$.

Using the resummed parton densities from Eq. (11), we can generate the ratios $F_2^{\text{res}}(x, Q)/F_2^{\text{NLO}}(x, Q)$.

The result of this test, picking $Q_0^2 = 100 \text{ GeV}^2$ is shown in Fig. 2, for the valence $F_2(x, Q)$ of the proton, with $x = 0.55, 0.65, 0.75$ and 0.85 . The NLO distributions were those of [2], and the inversion of moments was performed as in [11]. The effect of resummation is moderate for most Q . At small values of Q , and large x , the resummed structure function shows a rather sharp upturn. One also finds a gentle decrease toward very large Q [12]. We could interpret this difference as the *uncertainty* in the purely NLO valence PDFs implied by resummation.

From this simplified example, we can already see that the use of resummed coefficient functions is not likely to make drastic differences in global fits to PDFs based on DIS data, at least so long as the region of small Q^2 , of 10 GeV^2 or below, is avoided at very large x . At the same time, it is clear that a resummed fit will make some difference at larger x , where PDFs are not so well known. We stress that a full global fit will be necessary for complete confidence.

4. RESUMMED HADRONIC SCATTERING

Processes other than DIS play an important role in global fits, and in any case are of paramount phenomenological interest. Potential sources of large corrections can be identified quite readily in Eq. (2). At higher orders, factors such as $\alpha_s \ln^2 N$, can be as large as unity over the physically relevant range of z in some processes. In this case, they, and their scale dependence can be competitive with NLO contributions. Since they make up well-defined parts of the correction at each higher order, however, it is possible to resum them. To better determine PDFs in regions of phase

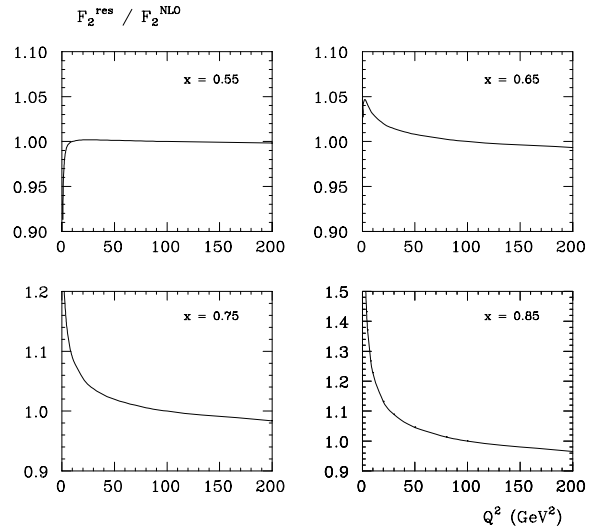


Figure 2. Ratio of the valence parts of the resummed and NLO proton structure function $F_2(x, Q^2)$, as a function of Q^2 for various values of Bjorken- x . For F_2^{res} , the ‘resummed’ parton densities have been determined through Eq. (11).

space where such corrections are important, we may incorporate resummation in the hard-scattering functions that determine PDFs.

The Drell-Yan cross section is the benchmark for the resummation of logs of $1-z$, or equivalently, logarithms of the moment variable N [9],

$$\hat{\sigma}_{q\bar{q}}^{\text{DY}}(N, Q, \mu) = \sigma_{\text{Born}}(Q) C_{\delta}^{\text{DY}} e^{E_{\text{DY}}(N, Q, \mu)} + \mathcal{O}(1/N). \quad (12)$$

The exponent is given in the $\overline{\text{MS}}$ scheme by

$$E_{\text{DY}}(N, Q, \mu) = 2 \int_{Q^2/\bar{N}^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} A(\alpha_s(\mu'^2)) \ln \bar{N} + 2 \int_{Q^2/\bar{N}^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} A(\alpha_s(\mu'^2)) \ln \left(\frac{\mu'}{Q} \right), \quad (13)$$

with A as in Eq. (10), and where we have exhibited the dependence on the factorization scale, setting the renormalization scale to Q . Just as in Eq. (10) for DIS, Eq. (13) resums all leading and next-to-leading logarithms of N .

It has been noted in several phenomenological applications that threshold resummation, and even fixed-order expansions based upon it, significantly reduce sensitivity to the factorization scale [13]. To see why, we rewrite the moments of the Drell-Yan cross section in resummed form as

$$\sigma_{AB}^{\text{DY}}(N, Q)$$

$$\begin{aligned}
&= \sum_q \phi_{q/A}(N, \mu) \hat{\sigma}_{q\bar{q}}^{\text{DY}}(N, Q, \mu) \phi_{\bar{q}/B}(N, \mu) \\
&= \sum_q \phi_{q/A}(N, \mu) e^{E_{\text{DY}}(N, Q, \mu)/2} \sigma_{\text{Born}}(Q) C_{\delta}^{\text{DY}} \\
&\quad \times \phi_{\bar{q}/B}(N, \mu) e^{E_{\text{DY}}(N, Q, \mu)/2} + \mathcal{O}(1/N). \quad (14)
\end{aligned}$$

The exponentials compensate for the $\ln N$ part of the evolution of the parton distributions, and the μ -dependence of the resummed expression is suppressed by a power of the moment variable,

$$\mu \frac{d}{d\mu} \left[\phi_{q/A}(N, \mu) e^{E_{\text{DY}}(N, Q, \mu)/2} \right] = \mathcal{O}(1/N). \quad (15)$$

This surprising relation holds because the function $A(\alpha_s)$ in Eq. (10) equals the residue of the $1/(1-x)$ term in the splitting function P_{qq} . Thus, the remaining N -dependence in a resummed cross section still begins at order α_s^2 , but the part associated with the $1/(1-x)$ term in the splitting functions has been canceled to all orders. Of course, the importance of the remaining sensitivity to μ depends on the kinematics and the process. In addition, although resummed cross sections can be made independent of μ for all $\ln N$, they are still uncertain at next-to-next-to leading logarithm in N , simply because we do not know the function A at three loops. Notice that none of these results depends on using PDFs from a resummed scheme, because $\overline{\text{MS}}$ PDFs, whether resummed or NLO, evolve the same way. The remaining, uncanceled dependence on the scales leaves room for an educated use of scale-setting arguments [14]. The connection between resummation and the elimination of scale dependence has also been emphasized in [15].

Scale dependence aside, can we in good conscience combine resummed hard scattering functions in Eq. (1) with PDFs from an NLO scheme? This wouldn't make much sense if resummation significantly changed the coefficient functions with which the PDFs were originally fit. As Fig. 2 shows, however, this is unlikely to be the case for DIS at moderate x . Thus, it makes sense to apply threshold resummation with NLO PDFs to processes and regions of phase space where there is reason to believe that logs are more important at higher orders than for the input data to the NLO fits.

At the same time, a set of fits that includes threshold resummation in their hard-scattering functions can be made [10], and their comparison to strict NLO fits would be quite interesting. Indeed, such a comparison would be a new measure of the influence of higher orders. A particularly interesting example might be to compare resummed and NLO fits using high- p_T jet data [3].

5. POWER-SUPPRESSED CORRECTIONS

In addition to higher orders in $\alpha_s(\mu^2)$, Eq. (1) has corrections that fall off as powers of the hard-scattering scale Q . In contrast to higher orders, these corrections require a generalization of the *form* of the factorized cross section. Often power corrections are parameterized as $h(x)/[(1-x)Q^2]$ in inclusive DIS, where they begin at twist four. In DIS, this higher twist term influences PDFs when included in joint fits with the NLO and NNLO models, and vice-versa [16–18]. As in the case with higher orders, such “power-improved” fits should be treated as new schemes.

6. CONCLUSIONS

The success of NLO fits to DIS and the studies of resummation above suggest that over most of the range of x , theoretical uncertainties of the NLO model are not severe. At the same time, to fit large x with more confidence than is now possible may require including the resummed coefficient functions.

Resummation is especially desirable for global fits that employ a variety of processes, such as DIS and high- p_T jet production, which differ in available phase space near partonic threshold. In a strictly NLO approach, uncalculated large corrections are automatically incorporated in the PDFs themselves. As a result, the NLO model cannot be expected to fit simultaneously the large- x regions of processes with differing logs of $1-x$ in their hard-scattering functions, unless these higher-order corrections are taken into account.

The results illustrated in the figures suggest that these considerations may be important in DIS with Q^2 below a few GeV^2 and at large x , where they may have substantial effects on estimates of higher twist in DIS. In hadronic scattering, large- N ($x \rightarrow 1$) resummation, which automatically reduces scale dependence, may play an even more important role than in DIS.

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